Probing Hadronic Structure with The Decay $\Delta \to Nl^+l^-$

Malcolm N. Butler

Department of Physics, Stirling Hall Queen's University Kingston, Canada, K7L 3N6

Martin J. Savage[†]

Department of Physics, 9500 Gilman Drive 0319 University of California, San Diego La Jolla, CA 92093-0319

Roxanne P. Springer

Department of Physics
Duke University, Durham, NC 27708-0305

Abstract

We compute the branching ratio for $\Delta \to Ne^+e^-$ and $\Delta \to N\mu^+\mu^-$ in chiral perturbation theory and find that both decays should be observable at CEBAF. With sufficiently low thresholds on the e^+e^- invariant mass a branching ratio of $\sim 10^{-5}$ may be observed for $\Delta \to Ne^+e^-$. For the $\Delta \to N\mu^+\mu^-$ decay mode we predict a branching ratio of 3×10^{-7} . The dependence of the M1 and E2 amplitudes on the momentum transfer will provide a useful test of chiral perturbation theory which predicts $\sim 20\%$ variation over the allowed kinematic range.

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[†] SSC Fellow

The radiative decays of the Δ and hyperon resonances can provide valuable insight into the nature of baryon structure and dynamics; a laboratory for probing nonperturbative QCD. These decays have recently been studied from the standpoint of heavy baryon chiral perturbation theory (HB χ PT) [1] [2] and from quenched QCD lattice gauge theory [3]. The SU(3) allowed transitions should be easily observable at CEBAF. The SU(3) forbidden modes, having branching ratios substantially smaller than those of the SU(3) allowed modes, will be on the verge of observability. In [2] the E2/M1 mixing ratio (ratio of reduced matrix elements for electric quadrupole to magnetic dipole radiation) was computed including final state interactions arising from soft pion rescattering. The mixing ratios were found to be large, with substantial imaginary parts.

Further information about the radiative vertex can be gleaned from a study of $\Delta \to Nl^+l^-$, where l is an electron or muon. The off-shell behaviour of this vertex can be determined by studying the decay rate of $\Delta \to Nl^+l^-$ as a function of the lepton pair invariant mass. In this work we will extend the calculations of ref. [1] and [2] to compute the rate for this process. Both $\Delta \to Ne^+e^-$ and $\Delta \to N\mu^+\mu^-$ should be observable at CEBAF; if sufficiently low invariant mass lepton pairs can be resolved then the branching fraction for $\Delta \to Ne^+e^-$ could be as large as $\sim 10^{-5}$.

We are working in the limit that the baryon mass, M_B , is much larger than any momentum transfer in the process, such as the pion mass, photon energy, etc. (for a review see [4]). In such a limit the four-velocity of the baryon, v_{μ} , becomes a good quantum number, as does its spin. In computing the matrix element for $\Delta \to Nl^+l^-$ it is most convenient to work in the frame where the lepton pair are back—to—back with invariant mass s. The M1 and E2 amplitudes, $A_1(s)$ and $A_2(s)$ respectively, have been computed at the one loop level in [1] and [2] and are shown in fig. 1. The M1 amplitude is dominated by a dimension five local counterterm that is fit to the decay rate for $\Delta \to N\gamma$, and by one-loop graphs involving kaons that give characteristic contributions of the form $\pi M_K/\Lambda_{\chi}^2$. On the other hand, the E2 amplitude is dominated by long-distance physics through pion loops contributing terms of the form $\log(M_{\pi}/\Lambda_{\chi})$. The extension to off–shell photons is straightforward, where now the s dependence of the kaon and pion loop graphs is retained. With these trivial modifications the results of [1] and [2] will be used with no further discussion.

We find that the spin averaged square of the matrix element for $\Delta \to N l^+ l^-$ is given by

$$\frac{1}{4}|\mathcal{M}|^2 = \frac{2e^2}{3} \frac{M_B^2}{s^2} (\Delta m^2 - s) \left[|A_1(s)|^2 \left(s(1 + \beta_l^2 \cos^2 \theta) + 4m_l^2 \right) + \frac{1}{5} |A_2(s)|^2 \left(4 \frac{s^2}{\Delta m^2} (1 - \beta_l^2 \cos^2 \theta) + 3s(1 + \beta_l^2 \cos^2 \theta) + 12m_l^2 \right) \right] , \quad (1)$$

where θ is the angle between the lepton momentum and the baryon momentum, Δm is the $\Delta - N$ mass difference, m_l is the lepton mass, and $\beta_l = \sqrt{1 - 4m_l^2/s}$ is the lepton velocity. The decay rate is then given by

$$\Gamma_{l^{+}l^{-}} = \frac{\alpha}{36\pi^{2}} \int_{4m_{l}^{2}}^{\Delta m^{2}} ds \frac{1}{s^{2}} \beta_{l} (\Delta m^{2} - s)^{3/2} \left[|A_{1}(s)|^{2} (s + 2m_{l}^{2}) + \frac{1}{5} |A_{2}(s)|^{2} \left(2\frac{s^{2}}{\Delta m^{2}} + 3s + 2m_{l}^{2} \left(3 + \frac{2s}{\Delta m^{2}} \right) \right) \right]$$
(2)

We have presented $d\Gamma_{l^+l^-}/ds$ as a function of the lepton pair invariant mass s in fig. 2. The contribution from $A_2(s)$ is much smaller than that from $A_1(s)$, as expected. Unfortunately, even deviations in the M1 angular distribution arising from the small admixture of the E2 component are too small to be measured experimentally and consequently this will not be a feasible method to independently determine the E2/M1 mixing ratio, $\delta_{\rm E2/M1}$. We have also presented the branching ratio for $\Delta \to Ne^+e^-$ as a function of the minimum detectable lepton pair invariant mass, $s_{\rm min}$, in fig. 3.

There is an uncertainty of approximately 20% in these results arising from uncertainties in the coupling constants used to determine $A_1(s)$ and $A_2(s)$. This issue is discussed in some detail in refs. [1] and [2]. Since the computation was done at the one-loop level there are $\sim 30\%$ systematic uncertainties arising from our neglect of terms higher order in the chiral expansion.

For $s_{\min} = 4m_l^2$ (the minimum value corresponding to both leptons being produced at rest), both $\Delta \to Ne^+e^-$ and $\Delta \to \mu^+\mu^-$ have branching ratios which will be easily observable at CEBAF; approximately 5×10^{-5} and 3×10^{-7} respectively. Obviously, $s_{\min} = 4m_e^2$ is unrealistically low due to insurmountable backgrounds from soft QED processes, but an observable branching ratio of $\sim 10^{-5}$ for $\Delta \to Ne^+e^-$ may not be unrealistic. We urge experimentalists planning to study the decay modes of the Δ to push their thresholds on the lepton pair invariant mass to as low a value as possible. This is crucial for making an accurate determination of the s dependence of the M1 amplitude.

In conclusion, we have computed the rates for $\Delta \to Ne^+e^-$ and $\Delta \to N\mu^+\mu^-$ in chiral perturbation theory and find that both modes should be easily observable at CEBAF. Measurement of the differential cross section for these decays will provide detailed invaluable information on the "formfactor" of the M1 amplitude. Chiral perturbation theory predicts only a slight variation, $\sim 20\%$, with respect to the invariant mass of the lepton pair over the entire allowed kinematic range arising from the kaon loops (the local counterterm has no s dependence) as shown in fig. 1. Determination of this s dependence would be a nice test of chiral perturbation theory.

We also note that a similar analysis could be done for $\Sigma^* \to \Sigma l^+ l^-$ and $\Xi^* \to \Xi l^+ l^-$, though they would be suppressed relative to real photon decay by the same amount as the $\Delta \to N l^+ l^-$ decay. Even more interesting would be a measurement of the s dependence of the SU(3) forbidden transitions (which do not receive a contribution from the local counterterm and consequently will have a much stronger s dependence than the SU(3) allowed transitions). Unfortunately, it seems unlikely that CEBAF will produce a large enough number of strange resonances to make such measurements feasible.

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Figure Captions

- Fig. 1. The M1 and E2 amplitudes as a function of momentum transfer s. The real part of the M1 amplitude dominates the rate and has $\sim 20\%$ variation over the entire range of kinematically allowed values for s.
- Fig. 2. Differential decay rates for the two processes $\Delta \to N e^+ e^-$ and $\Delta \to N \mu^+ \mu^-$.
- Fig. 3. The branching ratio for $\Delta \to Ne^+e^-$, as a function of the threshold invariant mass s_{\min} .





